# Reverse and intermediate segregation of large beads in dry granular media 

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#### Abstract

Mixtures of two types of glass beads have been sheared in a chute flow, in a half-filled rotating drum, and placed in a funnel to form a pile. In the three experimental devices, for small size ratios, there is a segregation of the large beads at the surface of the flowing phase (usual case), but for high size ratios (above about 5) the large beads segregate inside (reverse segregation). Precise measurements show that the segregation drives the large beads to an intermediate level inside the bed. In all devices, there is a continuous evolution of the location of the segregated beads from the surface to deep inside, when increasing the size ratio between the beads. The location of the segregated beads at intermediate levels is well defined both for high size ratios (above 5) and for very small size ratios (about 2), the level being very close to the surface in that case. The reverse and intermediate segregations are masked when using high fractions of large beads in the experiments. Their interpretation involves the high mass of the large particles balancing geometrical effects at a particular intermediate level inside the flowing layer.


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## I. INTRODUCTION

It is well known that when different particles are put into relative motion by shearing, flow, or vibration, they do not mix, and have the tendency to regroup in some areas. This segregation appears as soon as there are differences in size [1-3], density [4], shape, roughness [5], or angle of repose [6,7]. For particles of the same composition, the segregation of large and small beads has been investigated in numerous studies, and generally the large particles are found at the free surface of a medium composed of small particles (conversely small particles are found at the bottom of a group of large ones). In both cases, the large or small beads are simply squeezed out of the main bed, either at the surface or at the bottom, and the segregation seems to be independent of the relative volume fraction of the two species mixed together. This segregation has been explained by geometrical effects: as particles flow together, it is more likely for a small particle to find a large enough hole to fall inside than for a large one. The small particles go down, and consequently the large particles go up. This process has been called dynamical sieving. But this geometrical interpretation does not take into account the fact that some particles may push their neighbors to create some space and go downwards [8]. Furthermore, it has been argued that the large particles rolling on the surface see the small scale roughness of the bed of small ones, which can lead to an additional relative motion. This can be observed in the formation of a heap [9-12], where large particles roll on the side slopes and accumulate at the base of the heap, or in a half-filled rotating drum [2], where large particles go further than the small ones and stop at the periphery of the cylinder. In the same way, small particles are trapped in the roughness of a bed composed of large particles, which leads to a cylindrical central core in a heap [13] or to a central core of small particles in a rotating drum [3]. This surface segregation of the large beads can lead to stratifica-

[^0]tion patterns when the way of deposition is complex $[6,7,14]$. But once again, the geometrical interpretation of the process of segregation assumes that the bed of small beads is mechanically strong enough to support the weight of a large bead, and that large beads cannot push away the beads of the bed to go down inside.

The present study has been motivated by a volcanological application. Pyroclastic flow deposits are the result of avalanches of blocks of rocks, ashes and gas. These flows contain particles of various sizes and densities and usually exhibit patterns of segregation. In these natural systems, the particle size varies from one micron to a few meters [15]. We do not work with such a range, but we will consider the case of a large size ratio (from 1 to 40 ).

Except for a few studies on vibrated beds of granular media [16-19], most of the experimental studies have been carried out with small size ratios (for example, around 3 for chute flows [1], 1.6-3 for flows in a drum [3], and 3 for stratification in Hele-Shaw cell [20]). We are interested here in the influence of the size ratio when it is larger than 5, which also induces a high mass ratio between the beads. In this paper we show that these conditions will put the beads in a case where a large and, consequently, heavy bead can push away the small ones. The segregation leads then to patterns of reverse-segregation ["reverse" means that considering one single avalanche, the large beads are not segregated at the surface as it is usually the case (see Sec. II B 1 (case 2) or the end of Sec. II B 2 for more accurate definition)] and also to the surprising existence of segregated large beads gathered at an intermediate level inside the bed. These patterns exist in the deposit of a single avalanche, and the reverse relative positions of the large and small beads is not the result of a complex deposition of the flow as it is the case in the stratification described in the Hele-Shaw cell [7,20,14]. In our experiments, the location of the large beads in the deposit is directly related to those inside the flowing phase. To our knowledge, the segregation of large beads at an intermediate level inside the flow has not been described in a publication, although the "push-away" effect of a heavy bead was sug-


## assumed diameter of the beads

FIG. 1. Mean values $(*)$ of the diameters of "well-sorted" beads (measured with an optical microscope on a sample of 104 beads). Some beads are painted (red-violet-blue). Minimum ( + ), maximum ( + ), and standard deviations (errors bars) show about $10 \%$ relative spreading around the indicated diameter.
gested in a theoretical paper [8]. The present study is the first systematic experimental investigation of this new process of segregation. Furthermore, at these high size ratios, we expect that the segregation depends on the relative fraction of the two species: weak fractions of small particles can travel through the porous medium made by the large particles (for a diameter ratio larger than 6.46). On the other hand, for a large particle placed in a bed of small ones, no geometrical consideration can easily indicate that there will be a critical value of the size ratio at which the behavior would change. In some experiments, we will increase the volume fraction of the large beads in order to study the influence of the interaction between the large beads on the segregation behavior.

We focus our study on experimental setups where the particles are submitted to a shear motion as they are in any type of avalanche. Our flows are created in several ways: the release of a granular mixture from the top of a slope, the formation of a pile, or the rotation of a half-filled drum.

## II. EXPERIMENTS

The particles used are glass beads with diameters between $45 \mu \mathrm{~m}$ and 7.5 mm . The density of the glass is 2.5 (from 2.46 to 2.54 , with no systematic variation with the size). We sometimes color one of the two species by painting the beads uniformly. This process does not change their shape (checked with an optical microscope), but may slightly change the roughness of their surface, inducing a weak change in the angle of repose (from $24.7^{\circ}$ to $26.1^{\circ}$ for noncolored beads, and $28.6^{\circ}$ for colored beads). Nevertheless, we repeated some of the experiments, with colored and noncolored beads, and did not notice any difference in the behavior of both types of glass beads. In our experiments, the effect of the small difference of angle of repose is negligible compared to the effect of the large difference of size. Glass beads are either very well sorted beads, in order to work with "monodisperse" populations (Fig. 1); carefully sorted in a range of sizes, using sieves; or simply picked from the purchased bag (ranges of sizes indicated by an asterisk). In these latter cases, the diameter ratios are calculated with the mean


FIG. 2. The chute is 1 m long, and $\alpha$ is $26.5^{\circ}$. A 'homogeneous'" mixture (fine alternate layers of each type of beads) is placed in the box, and released by the opening of the gate. The deposit is cut perpendicular to the direction of the flow at various distances from the source (see Fig. 4).
diameter of each range. In order to minimize the electrostatic effect and the humidity cohesion effect between glass beads, the humidity in the laboratory room is held between $50 \%$ and $55 \%$, and special care is given to the experiments involving the smallest beads.

Three experimental devices are used in this study. In all cases, an initially homogeneous mixture of two types of beads flows in a slow frictional regime. We follow the behavior of the minority component (indexed 2) in a bed made of the majority component (indexed 1 ). A mixture is characterized by the diameter ratio $d_{2} / d_{1}$, and by the volume fraction of each type of beads.

## A. Flow along a slope

The chute is a $1-\mathrm{m}$-long, 6 - cm -wide channel inclined with an angle of about $26.5^{\circ}$ chosen to produce a frictional flow (Fig. 2). The bottom is made of a plastic sheet to which glass particles have been glued in order to obtain a rough surface. Glued particles are chosen to have the same size as the majority component of the mixture. At the top of the slope, a box contains 1 kg of the mixture composed of $10 \%$ of the minority species and $90 \%$ of the majority one. Due to the large quantities of particles required and lost in these experiments, we use particles within ranges of sizes. The starting box is filled by alternating fine layers of both species in order to obtain a homogeneous mixture at a large scale. At time 0, a gate is opened between the box and the chute, so that the mixture can flow with a mass flux of about $400 \mathrm{~g} / \mathrm{s}$. The flow is stopped when the channel becomes horizontal, or by a perpendicular wall usually located at 90 cm from the top (Fig. 2). We observe the surface deposit, then water the bed carefully and observe the internal structure by making some cross sections perpendicular to the direction of the flow. Deposits are between 2 and 5 cm thick, much thicker than the diameter of the large beads.

Many combinations of two classes of particles among the available beads have been used in this device, leading to different size ratios (from 0.09 to 44 ). Two types of segregation patterns are observed, depending only on the size ratio (Fig. 3): (1) the large beads cover the surface; and (2) the large beads are not at the surface, but inside the bed composed of small ones: there is a 'reverse segregation."

Case (1) appears for $d_{2} / d_{1}<4$ : small beads go down and large beads go up during the flow. We term this phenomenon the 'usual segregation', of large and small particles, which


FIG. 3. Experiments in a chute flow. Symbols indicate what pair of beads is involved in order to obtain the corresponding size ratio [ranges (*) or sizes indicated in the frame]; the vertical position on the graph shows their fraction in the mixture. For a size ratio lower than 4 , large beads segregate at the surface of the deposit, whether they are in the minority or the majority. For a diameter ratio above 5, 10\% of large beads are found inside the deposit. Experiments with a size ratio equal to 4.3 show a transitional behavior, as large beads are inside the deposit and at its surface [see Fig. 4(c)].
means that large beads are covering the surface. Working with $90 \%$ of large beads leads to fine particles forming a layer at the bottom of the cross sections [Fig. 4(a)]. For flows containing $10 \%$ of large beads, the deposit exhibits a surface layer which is composed of large particles. For the smallest size ratios (but still above 1), the segregation is very efficient, and not a single large particle is found underneath the surface layer [Fig. 4(b)]. For larger size ratios (but still lower than 4), a few large beads are found inside the bed, the great majority of the large beads being at the surface. For $d_{2} / d_{1}$ $<4$, the cases with $10 \%$ or $90 \%$ of large particles give two different points on the graph $\left[d_{2} / d_{1}\right.$ and $\left.1 /\left(d_{2} / d_{1}\right)\right]$ (Fig. 3) but correspond to the same physical process. We carried out a few more experiments with $50 \%$ of each type of beads to check that the value of the fraction of the two species does not influence the location of the segregated beads in three experiments $\left(d_{2} / d_{1}=1.75,2\right.$, and 3.5$)$. These experiments also lead to large beads at the surface, and small beads at the bottom.

Case (2) is more surprising. It appears for large size ratios ( $d_{2} / d_{1}$ greater than 5 ) there are no large beads on the surface of the bed. The cross sections in the deposit show that the large beads are randomly distributed through the whole thickness, except in the area near the surface [Figs. 4(d) and 4(e)]. However, this is not the result of a true mixing because there are no large beads on the surface or near the surface, nor at the bottom. We call this 'reverse segregation,'" for there is not a single large bead on the surface. For larger size ratios, we notice that the large beads are located closer to the bottom [Fig. 4(e)]. Moreover, one experiment made with $d_{2} / d_{1}=4.3$ shows a transitional behavior: the large beads are found both on the surface and inside the deposit [Fig. 4(c)], leading to a homogeneous bed. These two last points suggest that there is a continuous evolution of the location of the segregated large beads from the surface (for $d_{2} / d_{1}<4$ ) to a deeper interior level when increasing the diameter ratio.

For one size ratio corresponding to case (2) $\left(d_{2} / d_{1}\right.$ $=8.6$ ), some experiments were performed with chute of different lengths: 30,60 , and 90 cm . No difference has been observed between the three lengths of the chute, indicating


FIG. 4. Cross sections in the deposits obtained in the chute flow experiments ( 6 cm wide). (a) $d_{2} / d_{1}=0.5$ : layer of $300-400-\mu \mathrm{m}^{*}$ blue beads under a bed of $600-800-\mu \mathrm{m}^{*}$ beads. (b) $d_{2} / d_{1}=2$ : layer of $3-\mathrm{mm}$ blue beads at the surface of a bed composed of $1.5-\mathrm{mm}$ beads. (c) $d_{2} / d_{1}=4.3$ : beads of 3 mm and of $600-800$ $\mu \mathrm{m}^{*}$ constitute an homogenous deposit. (d) $d_{2} / d_{1}=8.6$ : large beads ( 3 mm ) are all inside the deposit ( $300-400 \mu \mathrm{~m}^{*}$ ), and the surface is free of large beads. (e) $d_{2} / d_{1}=44$ : large beads ( 3 mm ) are inside the deposit (made of $45-90-\mu \mathrm{m}^{*}$ beads), and seem to be located deeper than for a smaller size ratio.
that a stationary state is reached almost immediately.
The segregation is completely different in experiments having a size ratio less than 0.2 , with those having a size ratio greater than 5 (with the same pair of beads). $10 \%$ of large particles are located at intermediate levels [case (2)], while $90 \%$ of large particles are at the surface [case (1)]. Because a few fine particles percolate through a network composed of large beads, we expect this result still to be valid when the size ratio decreases to zero. This means that a majority of large particles is always at the surface, although a minority of large beads can be either at the surface or inside the bed depending on the value of the size ratio. We conclude that for large size ratios (larger than 5), the segregation depends on the volume fraction of the two species.

Briefly, increasing the size ratio results in a lowering of the level of segregated large beads: they tend to segregate at the top (for $1<d_{2} / d_{1}<4.3$ ), then to reach an intermediate level (for $d_{2} / d_{1}$ above 5), and then to sink toward the bottom for larger $d_{2} / d_{1}$. At this stage, we did not quantify the position of the large beads in the cross sections. In fact, direct


FIG. 5. The funnel is filled with alternate layers of each type of beads to obtain a "homogeneous'" mixture of the two components. The pile forms on a rough surface. On the drawing, the segregation drives the large beads into a ring at the bottom of the pile slope.
observations of the flow and of the internal structure of the deposit show that the deposit is the result of the deformation of the main flow when it stops. The bottom layer is not modified by the deformation, and the surface of the flow stays on the surface of the deposit when it stops: there is no doubt in diagnosing the usual segregation of fine beads at the bottom (for $d_{2} / d_{1}<1$ ) or large beads at the top (for 1 $\left.<d_{2} / d_{1}<4\right)$. On the contrary, the intermediate area is strongly affected by the deformation of the flow when it comes to a halt. In the case of a reverse-segregation pattern, we do not have the opportunity to quantify the exact vertical location of the large beads during the flow. For this reason, we used another experimental device to study the evolution between the usual segregation of the large particles [case (1)] and the reverse-segregation of larger particles [case (2)].

## B. Formation of a heap

When a mixture of two types of beads is placed into an hourglass or a funnel, the heap formed beneath exhibits features of segregation [21,12]. Very little segregation occurs in the hopper, and it is focused at the very beginning of the flow [22]. In fact, the segregation results from phenomena taking place in the flow along the pile slopes. First, there is a segregation in the flowing layer by dynamical sieving. Second, on the slopes of the pile, the large particles roll on a relatively smoother surface than the small ones, and go further than the small ones [23]. Due to the rolling of some beads or due to the fact that the upper surface of the flow goes further than its bottom (for an example, see Ref. [24]), the particles of the surface of the flowing phase end up in the lower part of the surface of the pile and accumulate in a ring [see Fig. 5 or 6(a)]. For vanishing fluxes, only the rolling process takes place, but in our experiments the funnel has an output aperture of 1.2 cm in diameter, inducing a flux range between 40 and $60 \mathrm{~g} / \mathrm{s}$ depending on the type of particles used. This means that the flow is continuous along the slope of the pile in the first stage, then proceeds as successive avalanches when the pile becomes larger, but never as individual beads rolling along the slope.

We place a homogeneous mixture of the two components in the funnel by putting successive fine layers of each type of beads. Large beads are $3-\mathrm{mm}, 1.5-\mathrm{mm}$, and $710-\mu \mathrm{m}$ colored glass beads, and represent a fraction varying from $1 \%$ to $50 \%$. Small beads are either well-sorted beads or comprise a


FIG. 6. Top view of the segregation in the piles. Dark beads are $3-\mathrm{mm}$ large beads ( $20 \%$ of the mass) mixed with (a) $710-\mu \mathrm{m}$, (b) $500-\mu \mathrm{m}$, (c) $425-\mu \mathrm{m}$, (d) $300-\mu \mathrm{m}$, (e) $150-250-\mu \mathrm{m}^{*}$, and (f) $45-90-\mu \mathrm{m}^{*}$ small beads. For small size ratios, the large beads form a ring around the bottom of the pile $[(a)$, (b), and (c)]. For large size ratios $[(\mathrm{d}),(\mathrm{e})$, and (f)], the large beads are inside the pile, and the ring is composed of small white particles. We choose the breaking of this ring [between pictures (c) and (d)] as a criterion for a limit between the usual and the reverse-segregation patterns. The scale is around 25 cm long.
range of sizes (see the figure captions). For most of the experiments, 800 g of small beads are placed in the funnel, combined with a known mass fraction of large particles. Experiments with a fraction larger than $25 \%$ are made with a smaller amount of small particles for convenience. The pile develops on a rough surface located 5 cm beneath the funnel (the final pile is around 4.5 cm high), which induces impacts at the very beginning of the flow, but does not have any effect on the final figure of segregation. This was checked by keeping the funnel $\sim 1 \mathrm{~cm}$ above the summit of the heap by hand during its growth.

## 1. Variation of the diameter ratio

A first series of experiments was carried out to explore the effect of the diameter ratio on the segregation of a given fraction of $3-\mathrm{mm}$ large blue beads among small ones. For small size ratios, the large beads are mainly located on the surface of the heap, and form a ring around the base of the pile [Fig. 6(a)]. This corresponds to the usual segregation of the large beads at the surface of the heap [case (1)]. For large
size ratios, we do not find any large beads on the surface or in the ring area: the large beads are embedded in the pile, which corresponds to reverse segregation [case (2)] [Fig. $6(\mathrm{f})$ ]. The evolution between these two patterns is progressive with the variation of the size ratio [Figs. 6(a)-6(f)]. Although the evolution of the segregation pattern is continuous, it is convenient to define a limit between usual- and reverse-segregation patterns. As a criterion, we choose the formation of a ring of large particles or small particles, because the intermediate case [between Figs. 6(c) and 6(d)] seems to correspond to the more homogeneous pile. Corresponding structures inside the pile are as follows. For usual segregation, the large beads are found in a fine layer at the bottom of the pile, because the growth of the pile covers the previous rings composed of large beads. For reverse segregation, the large beads are located everywhere inside the pile, except at its bottom, which is purely composed of small beads.

During the growth of the pile, the flowing layer is thicker than a large bead diameter near its summit, because we do not see the large beads totally embedded in the flowing phase. For geometrical reasons, the thickness of the flow decreases along the sides of the pile. In the case of reverse segregation, after some traveling, the top of some large beads comes into view emerging from within the flowing layer. We observe that the beads are not going up toward the surface during the flow, as they do in the case of usual segregation, but that they move embedded in the small beads. The occurrence of the top of the large beads coming into view happens more or less far from the top of the heap, presumably a consequence of their vertical location in the flowing layer or their ability to go down (or up) during the flow. These large beads end up stopping on the slope and do not continue to roll individually, even in the case where they are almost 'on the surface" due to the extreme thinning of the flowing layer. Consequently they do not regroup in a ring at the base of the pile [Figs. 6(e) and 6(f)]. For very large size ratios, this tendency is enhanced, and large particles tend to group in the central part of the pile beneath the feeding funnel. All these observations made on the heaps suggest a continuous evolution of the location of the segregated beads with the size ratio.

## 2. Variation of the fraction of large beads

A second series of experiments was conducted with different fractions of large beads. The same evolution can be observed for a fixed diameter ratio and a variation of the volume fraction of large beads from $1 \%$ to $50 \%$ (Fig. 7). At high fractions, the large beads are on the surface of the pile and form a ring. At small fractions, the large beads are inside the pile. Consequently, for each size ratio, we can again define the limit between the two patterns of segregation, using the same criterion. For fractions above this limit, there is the usual segregation, whereas under this limit there is reverse segregation. The limit shifts toward large fractions for high diameter ratios (Fig. 8). It is difficult to define a limit for small size ratios (under about 3), because this would correspond to very small fractions: there are not enough large beads to observe a well-marked ring, and our criterion is no longer applicable. Nevertheless, we can observe the individual behavior of the large beads. For a very low size ratio


FIG. 7. The evolution of the surface of the pile with increasing fractions of large beads is similar to that in Fig. 6. White beads are $300-\mu \mathrm{m}$ beads combined with (a) $15 \%$ and (b) $35 \%$ of dark blue 3-mm large beads. At low fractions, large beads are embedded in the pile. At high fractions, the large beads accumulate in a ring. For the scale see Fig. 6.
(1.5), the large beads are on the surface, but they do not roll very easily on the surface because the roughness is not very low in comparison with their size: they do not build a very well-marked ring. For higher size ratios (but below about 3), the large beads roll and reach the ring area. When the size ratio is larger than 3 , the large beads are partially embedded in the bed of small beads, and do not move easily along the slope.

Changing both the diameter ratio and the fraction of large beads can lead to similar final pictures of the pile surface. As an example, the surface of the pile in Fig. 7(a) is intermediate between those in Figs. 6(d) and 6(e). Looking at photos of the surface of the piles all together, without knowing to which experiments they refer, we can form them into groups with the same pattern. In Fig. 8, experiments with the same pattern are joined by lines. Our criterion for a limit between the two fields of segregation is one of these lines. With that criterion, and for a fraction of $10 \%$ of large beads, the limit corresponds to a diameter ratio $d_{2} / d_{1}$ of about 5 , which is compatible with the results obtained with the experiments in the chute. The evolution of all these lines shows that, for a higher size ratio, a greater fraction of large particles is required to obtain the same surface picture of the pile: we deduce that a larger number of large beads are deep inside.


FIG. 8. Segregation in the piles. Large beads are 3 mm , small beads are well-sorted beads (see Fig. 1), except for $150-250 \mu \mathrm{~m}^{*}$, $70-110 \mu \mathrm{~m}^{*}$, and $45-90 \mu \mathrm{~m}^{*}$ (*see text). Symbols indicate usual segregation $(*)$, and reverse segregation $(+$ ) according to the criterion chosen. $(\mathbf{\Delta})$ and $(\times)$ indicate experiments very close to the limit (usual and reverse, respectively). The lines link piles whose surfaces look identical. For a large diameter ratio, a high fraction of large beads is required for the large beads to form a ring, suggesting that the beads are on average located deeper than for a smaller size ratio. For fractions as large as $50 \%$, large beads always form a ring.

This point suggests that the large beads are on average "deeper"' for larger size ratios. The location of the large beads seems to evolve continuously from the top toward the bottom of the flowing layer when increasing the size ratio.

These results are compatible with the well-known case of a heap containing $50 \%$ of each type of particles. In this case, the coarse particles cover the lower part of the surface of the pile and form a ring, even for a size ratio as large as 44 (Fig. 8).

Some experiments have been made with $1.5-\mathrm{mm}$ or $710-\mu \mathrm{m}$ large beads. They were carried out with the same flux and the same size pile. Once again, the two patterns of segregation (usual and reverse) are obtained (Fig. 9). The three series of results are roughly the same. Except perhaps for the results at very small fractions, we can conclude that the limit between the fields of segregation seems to depend on the size ratio and not on the value of each diameter. For very small fractions, it is difficult to define a ring due to the small number of large beads. For the same fraction, the "smaller" large beads are far more numerous than the $3-\mathrm{mm}$ large beads, and this fact can explain the different results obtained at small fractions. Moreover, reducing the size of the beads is analogous to studying a larger pile: the transition between the two types of segregation does not depend on the size of the pile. This agrees with the fact that the segregation pattern looks the same during the growth of the pile-except at the very beginning, when impacts on the rough surface are disturbing the flow.

In conclusion, pile experiments exhibit the same two types of segregation of large particles as the chute experiments, the "transition" occurring for similar fractions and size ratios. The main difference between these experiments is the thickness of the flowing layer. Due to a larger flux, the


FIG. 9. Comparison between three types of large beads $(\boldsymbol{)} 3$ mm , (■) 1.5 mm , and ( $\mathbf{(}) 710 \mu \mathrm{~m}$ in diameter; open symbols correspond to a white ring composed of small particles (reverse segregation), and plain symbols to a ring of large beads (usual segregation). The line is the limit obtained from the experiments done with $3-\mathrm{mm}$ large beads (Fig. 8). Experiments with $710-\mu \mathrm{m}$ large beads have been carried out with well-sorted beads or with the ranges $300-400 \mu \mathrm{~m}^{*}, 180-212 \mu \mathrm{~m}, 150-250 \mu \mathrm{~m}^{*}, 70-110$ $\mu \mathrm{m}^{*}$, and $45-90 \mu \mathrm{~m}^{*}$. Those with $1.5-\mathrm{mm}$ large beads have been carried out with well-sorted beads, or with the ranges 180-212 and $150-180 \mu \mathrm{~m}$. For $3-\mathrm{mm}$ large beads experiments, see Fig. 8.
flow in the chute is much thicker and the large beads, if moving upward, may not have enough time to reach the surface. However in the case of pile formation, the thickness is much smaller, and cannot prevent large beads from reaching the surface. We then deduce that the large beads are preferentially going to an intermediate level inside the bed rather than being pushed to the surface.

Moreover, the pile experiments show how the segregation pattern can be sensitive to the fraction of beads. When increasing the fraction of the large beads, reverse segregation shifts more or less rapidly to usual segregation (depending on the size ratio), revealing that the segregated large beads are located more or less deeply at an intermediate level. In all cases, the large beads gather at a preferential level in the flowing layer: at the top (surface), at an intermediate level (more or less deep inside), or at the bottom; hence a variable number of large particles are shown at the surface. All these observations suggest a continuous evolution instead of an abrupt limit between usual and reverse segregation. We propose to define three types of locations of segregated beads and the associated segregations. If the large beads are on the surface of the flow, we call it 'up segregation''; if they are at the bottom, it is "down segregation'" and if they are somewhere inside the bed, we speak about "intermediate segregation." With these definitions, the usual segregation, which corresponds to large beads seen at the surface, includes the up segregation and the highest levels of the intermediate segregation; the reverse segregation corresponds to the down segregation and the levels of intermediate segregation deep enough so that the large beads are not visible at the surface.


FIG. 10. $r_{i}$ is the distance from each large bead to the center of the drum, and $r$ the mean value of all the $r_{i}$ for each experiment. The averaging is done with about 30 measurements for experiments involving $3 \%$ of 3-mm large beads, with about 90 measurements for $16.6 \%$; with 200-500 measurements for higher fractions; and with only about ten measurements for experiments carried out with $1 \%$ or $0.5 \%$ of large beads, in relation with the number of large beads present in the drum in each case.

## C. Rotation in a half-filled drum (or disk)

The drum is half-filled and turns around its axis, which is placed horizontally, and the flowing layer is located on the free surface. Two processes take place in a drum: radial segregation $[2,3]$ and axial segregation $[25,26]$. Here we study radial segregation: up segregation in the flowing layer pushes the beads as far as possible during the flow, and the beads are then located at the periphery of the drum; down segregation groups the beads near the center of the drum, as they sink as soon as they are injected into the flow. $R$ is the radius of the cylinder, and $r$ the mean distance from the considered beads to the center. $r / R$ quantifies the location of the segregated beads and varies from 1 (up segregation) to 0 (down segregation), $0<r / R<1$ corresponding to an intermediate segregation (Fig. 10).

The drum is 42 mm long, and 48.5 mm in diameter, made of steel (caps) and glass, and half-filled with the homogeneous mixture of small and large beads. The rotation is obtained manually by rolling the drum on a plane. Rotation speeds are around $0.06 \mathrm{~s}^{-1}$ (from 0.04 to $0.09 \mathrm{~s}^{-1}$ ), which corresponds to a continuous flow on a plane free surface. The beads are pushed toward the surface of the flow by the rotation in the first half of their travel in the flowing phase. Contrary to the case of the pile experiments, where impacts could drive the large beads deep into the flowing layer, we could consider that the initial location of a bead is on the whole at the surface of the flow. After few revolutions, we dip the drum into water. It is then possible to open it and to make several cross sections perpendicular to the axis of the drum.

## 1. Variation of the diameter ratio

A first series of experiments was made with small beads (ranges indicated in Figs. 11 and 12) combined with $16.6 \%$ of large beads ( $710 \mu \mathrm{~m}$ or 3 mm in diameter). Once again, depending on the size ratio, we observe the up, down, and intermediate segregations. During the experiments, the large


FIG. 11. View of the cross sections inside the drum for two diameter ratios (the drum is 4.85 cm in diameter). The dark beads are $710-\mu \mathrm{m}$ beads, representing $16.6 \%$ of the total mass in a bed composed of (a) $500-\mu \mathrm{m}$ and (b) $45-90-\mu \mathrm{m}^{*}$ small beads. The segregation evolves from usual segregation at the periphery of the drum to a homogeneous bed.
beads roll on the surface for small size ratios or rapidly sink for high size ratios. For very large size ratios, they start to move along the slope before reaching the surface, and flow embedded in a layer of small beads.

Experiments with $710-\mu \mathrm{m}$ large beads show axial and radial segregations. For small size ratios (under 2.3), the large


FIG. 12. Some experiments done with $16.6 \%$ of 3-mm large beads in the drum. For each experiment, all the cross sections are added on the same drawing. The location of the large beads evolves from the usual segregation at the periphery to a homogeneous bed when increasing the diameter ratio, and then to a gathering into a central core for very large size ratios. Small beads are (a) 2 mm , (b) 710-850 $\mu \mathrm{m}$, (c) $425-500 \mu \mathrm{~m}$, (d) $300-400 \mu \mathrm{~m}^{*}$, (e) $180-212$ $\mu \mathrm{m}$, and (f) $45-90 \mu \mathrm{~m}^{*}$, and the corresponding diameter ratios are $1.5,3.8,6.5,8.6,15.3$, and 44.


FIG. 13. All the cross sections added on the same drawing for some experiments with $3 \%$ of $3-\mathrm{mm}$ large beads. The small beads are (a) 1.5 mm , (b) $850 \mu \mathrm{~m}$, (c) $600 \mu \mathrm{~m}$, (d) $500 \mu \mathrm{~m}$, (e) $425 \mu \mathrm{~m}$, (f) $300 \mu \mathrm{~m}$, (g) $212 \mu \mathrm{~m}$, (h) $150 \mu \mathrm{~m}$, and (i) $90 \mu \mathrm{~m}^{\#}$ ( ${ }^{\#}$ done with $1 \%$ of $3-\mathrm{mm}$ beads). The location of the segregated beads evolves continuously from the periphery to the center when increasing the diameter ratio, but is always well marked, drawing a half circle inside the drum.
beads are at the surface, and at the periphery of the drum [Fig. 11(a)]. For size ratios from 2.8 to 4.3, the large beads are located more or less near the periphery. For very large size ratios (above 7.8), large beads are uniformly distributed inside, except in fine layers near the periphery and at the surface [Fig. 11(b)]. The transition toward this "homogeneous bed" is progressive for size ratios between 4 and 7 . The same patterns with the same limits are observed in the experiments involving 3-mm large beads, indicating that the process depends on the size ratio and not on the diameter of the small beads (Fig. 12).

The axial segregation is well marked for experiments with $710-\mu \mathrm{m}$ large beads, and shows two types of features. When large beads are on the surface, they accumulate preferentially in two surface bands near the caps and form also two layers touching the caps inside the bed. For a reverse segregation of the large beads, the small beads constitute the whole surface and the two layers touching the caps. The link between the axial and radial patterns corresponds to features seen with magnetic resonance imaging technique [27]. This axial segregation prevents us from precisely studying the radial segregation because the volume fraction of large beads is not constant along the axis. We work preferentially with $3-\mathrm{mm}$ large beads, for which the axial segregation is not well marked.

A second series of experiments have been carried out with $3 \%$ of $3-\mathrm{mm}$ large beads. Small beads are generally well sorted beads (Fig. 1). Figure 13 shows, for some experiments, all the cross sections (5-12 sections by experiment) added on the same drawing. For each size ratio, the large beads gather on a circle at a preferential distance from the center $r$ : there is a segregation. This circle moves from the periphery to the center when increasing the diameter ratio. The 'homogeneous'" character of the experiment made with $16.6 \%$ of large beads can be interpreted as the large beads spreading around this circle as a consequence of the interac-


FIG. 14. Segregation of $3 \%$ of $3-\mathrm{mm}$ large beads in smaller ones (from 2 mm to $68 \mu \mathrm{~m}$ ) in the rotating drum. The symbols indicate the mean $(\bigcirc)$, minimum $(+)$, and maximum $(+)$ values, and the standard deviations (represented as errors bars) of the distributions of the distances $r_{i}$. Open symbols $(\bigcirc)$ correspond to experiments where a range of sizes was used instead of well-sorted beads: 710$850 \mu \mathrm{~m}, 212-250 \mu \mathrm{~m}, 150-180 \mu \mathrm{~m} ; 70-110 \mu \mathrm{~m}^{*}$, and $45-90$ $\mu \mathrm{m}^{*}$. Symbols $(\diamond)$ correspond to experiments done with only $1 \%$ of large beads. Standard deviations are always small, indicating that there is a segregation. For small diameter ratios, the large beads are stacked at the periphery of the drum $\left(r / R_{\max }=0.938\right)$; hence there is up-segregation. For large size ratios, the half circle of coarse beads drifts continuously toward the center, which corresponds to down-segregation. For any other cases, there is intermediate segregation at $r, 0<r / R<1$.
tion between the large beads, and is not the result of a true process of mixing.

For each experiment, statistics have been calculated on the spatial coordinates of the large beads (Fig. 14). Standard deviations of the distributions of distances from each bead to the center of the drum $r_{i}$ (represented as error bars) are always small, indicating the existence of the segregation. The mean distance $(r)$ for the group of large beads evolves continuously from an up segregation $(r=R)$ to a down segregation $(r=0)$, when the size ratio increases. The maximum value of $r / R$ is equal to 0.938 , corresponding to the case where all the large beads are touching the periphery. The minimum value of $r / R$ is not equal to 0 because of the thickness of the flowing layer at the free surface. We assume that this thickness is at least as big as a large bead, and do not take into account the beads located at less than 3 mm from the surface during the measurements.

The segregation is slightly more efficient for a small size ratio than for a large one. The lack of space, or the shortening of the distance between the large beads, when they are all gathered on a small circle near the center, could explain the dispersion of the data. In experiments made with $1 \%$ of large beads, the standard deviation is slightly smaller in the experiment using small beads of $90 \mu \mathrm{~m}$ (Fig. 14). Moreover, there is a shift of the mean value to a smaller value of $r / R$ (also described in Sec. II C 5). The shift disappears when beads are gathered in larger circles (for small beads of 212 and 425 $\mu \mathrm{m})$. We deduce that when the large beads are close to each other, they begin to interact and modify the segregation.

The main result of this series of experiments is the clear visualization of the segregated beads at an intermediate level.

The segregation is always well marked for every intermediate location $r / R$ of the segregated large beads, and there is a continuous evolution of the location with the size ratio. This means that the segregation does not act by squeezing alien particles out of the bed, putting them neither at the surface ( $r=R$ ) nor at the bottom $(r=0)$. Large particles are located at an intermediate level inside the bed, corresponding to an intermediate stopping distance $r(0<r<R)$.
$r / R$ varies continuously with the diameter of the small beads (Sec. II C 1), and does not seem to be affected by a small variation of the speed of rotation (Sec. II C 2). Consequently, $r$ does not correspond to a local change in the characteristics of the flow, as for example, a local change of the slope of the free surface or of the geometry of the flowing layer. Moreover, $r / R$ is the same when changing the small beads, keeping $d_{2} / d_{1}$ constant (Sec. II C 3), or changing $R$ (Sec. II C 6): the flowing layer is not determined by the small beads. We deduce that $r / R$ corresponds to a "vertical" location in the flowing layer at the time when the bead stops (vertical meaning here perpendicular to the free surface). There are two possibilities. First, the large particle is in relative vertical motion compared to the flow. This relative motion would be interrupted when the flowing layer stops and the segregation process would not be fully achieved. Following this hypothesis, in all the experiments with $r / R$ larger than a certain value (for example 0.5), the large beads would go up to the surface (and $r=R$ ); conversely, in experiments with $r / R$ less than this value, they would go down to the bottom ( $r=0$ ). Thus, as all our experiments have been carried out in the same drum (i.e., flowing layers stopping at the same stage), $r / R$ would measure the "speed" of sinking or going up relative to the flowing layer. Second, the large bead reaches an equilibrium level within the flowing layer, and moves with the flow at this fixed vertical location. In the flowing phase stream lines are straight, and each intermediate vertical level stops at a different length. $r / R$ then measures the vertical coordinate of a large particle. That second explanation seems more likely because, while a large bead is moving, we observe that its vertical location in the flow looks constant: a fixed fraction of the large beads is visible above the free surface of the flow, indicating that a vertical equilibrium level is held during the travel of the large beads at the surface.

## 2. Variation of the speed of rotation

For two cases, small beads of $710 \mu \mathrm{~m}$ or within the range $212-250 \mu \mathrm{~m}$ combined with $3 \%$ of $3-\mathrm{mm}$ large beads, we varied the speed of rotation by a factor of about 10 . The low speeds correspond to the regime where successive avalanches take place (about $0.02 \mathrm{~s}^{-1}$ ). The high speeds correspond to the point where the free surface becomes visually curved [26] (about $0.2 \mathrm{~s}^{-1}$ ). Surprisingly, the results are very similar (Fig. 15). Our experimental device does not allow a fine control of the speed, and it is welcome that the speed does not have any influence on the results in the range of speeds where we did the experiments.

## 3. Changing both types of beads

A series of experiments has been carried out keeping the size ratio constant, equal to 10 , and changing both type of


FIG. 15. The speed of rotation of the drum does not have any effect on the location of the segregated beads in the range of speed where we did the experiments (from 0.04 to $0.09 \mathrm{~s}^{-1}$ ). The large beads are 3 mm in diameter, and represent $3 \%$ of the total mass; the small beads are ( $\mathbf{\nabla}) 710 \mu \mathrm{~m}$ and $(\bullet) 212-250 \mu \mathrm{~m}$ in diameter. $(+)$ are minimum and maximum values, and errors bars represent the standard deviations of the distributions of $r_{i}$.
beads (from $710 \mu \mathrm{~m}$ to 5 mm , considering the large ones). $r / R$ does not change when the diameters of small beads varies (Fig. 16). If we assume that the geometry of the flow does not depend on the type of small beads used, this indicates that the relative vertical position of a large bead within the flowing layer depends only on the size ratio.

## 4. Variation of the time of rotation

For an intermediate case, $500-\mu \mathrm{m}$ beads combined with $3-\mathrm{mm}(3 \%)$ beads, we vary the number of revolutions of the drum. No evolution of $r / R$ is observed (Fig. 17), suggesting that the convergence to a steady state is very rapid (less than one revolution), and only the values of the standard deviation


FIG. 16. Nondependence of the location of the segregated beads with the type of beads used. Symbols indicate mean values ( minimum and maximum values $(+)$ and standard deviation (errors bars). Small beads are $500 \mu \mathrm{~m}, 400 \mu \mathrm{~m}, 300 \mu \mathrm{~m}, 200 \mu \mathrm{~m}, 150 \mu \mathrm{~m}$, and $45-90 \mu \mathrm{~m}^{*}$, and corresponding large beads are $5 \mathrm{~mm}, 4 \mathrm{~mm}$, $3 \mathrm{~mm}, 2 \mathrm{~mm}, 1.5 \mathrm{~mm}$, and $710 \mu \mathrm{~m}$, leading to a constant size ratio equal to 10 .


FIG. 17. The location of the segregated beads does not depend on the number of revolutions of the drum. The convergence towards stationary state is very rapid (less then one revolution). There are $3 \%$ of $3-\mathrm{mm}$ beads in a bed of $500-\mu \mathrm{m}$ beads. Symbols indicate mean values () , minimum and maximum values $(+)$, and standard deviation (errors bars).
shows a little focusing of the segregation during the first revolution. This agrees with the study of Cantelaube and Bideau [3] that shows that the segregation pattern reaches its steady state in less than one revolution. As our experiments were carried out with three revolutions, our measurements concern the steady state.

## 5. Variation of the fraction of large beads

The main influence of the fraction is that the focusing of the segregation in a circle is masked for high size ratios and the deposit looks homogeneous (Figs. 11 and 12). Interaction between the large beads is also obvious from the standard deviations of the $16.6 \%$ experiments, which are larger than those of experiments with $3 \%$ of large beads (Fig. 18). As a


FIG. 18. Segregation of $16.6 \%$ of 3-mm large beads in a bed of small ones. Symbols indicate mean values $(\boldsymbol{)}$, minimum and maximum values ( + ), and standard deviations (errors bars) of the distributions of the distances $r_{i}$ from the large beads to the center of the drum. Small beads are $2 \mathrm{~mm}, 710-850 \mu \mathrm{~m}, 600-800 \mu \mathrm{~m}^{*}, 425-$ $500 \mu \mathrm{~m}, 300-400 \mu \mathrm{~m}^{*}, 180-212 \mu \mathrm{~m}, 70-110 \mu \mathrm{~m}^{*}$, and $45-90$ $\mu \mathrm{m}^{*}$.


FIG. 19. Influence of the fraction of large beads on the segregation in the rotating drum ( $3-\mathrm{mm}$ and $212-\mu \mathrm{m}$ beads). Symbols indicate mean values $(\boldsymbol{)}$ ) and standard deviations (errors bars). The two curves are the maximum and minimum values of $r / R$, taking into account of the volume occupied by the fraction, and considering that the whole group of large beads gather either at the periphery or at the center. A homogeneous bed corresponds to $r / R$ $=0.66$. Small fractions correspond to reverse segregation, whereas high fractions lead to usual segregation.
consequence of this steric effect, $r / R$ does also not reach its maximum ( 0.938 ) for small size ratios: some pockets of large beads stacked at the periphery prevent that all the beads touch the wall. For high size ratios, $r / R_{16.6 \%}$ is larger than $r / R_{3 \%}$; however, in that case, we observe that the center of the drum is still not filled with the large beads. The steric effect is not able to explain the difference between these values of $r / R$. Large beads genuinely segregate at a shallower level in experiments with higher fractions than in experiments with a smaller fraction of large beads.

A series of experiments investigates the influence of the fraction of large beads using small beads of $212 \mu \mathrm{~m}$ combined with large beads of 3 mm in diameter, whose fraction varies from $0.5 \%$ to $50 \%$ (Fig. 19). Increasing the fraction leads to a wider distribution, and to a shift of $r / R$ from 0.45 (reverse segregation) to 0.74 with a large spreading so that many large beads are stacked at the periphery (usual segregation). If the beads just filled the space around the initial value ( 0.45 ), $r / R$ would be equal to 0.45 , and would then tend to 0.66 (homogeneous bed) or would follow the $r / R$ minimum curve (Fig. 19). $r / R$ minimum (or maximum) is calculated taking account of the volume occupied by the fraction, and considering that the whole group of large beads gather at the center (or at the periphery). But, for high fractions, $r / R$ is about 0.74 which indicates that many more large beads are close to the periphery than to the center; the large beads segregate closer to the surface when they are present at high fractions.

## 6. Segregation in a two-dimensional disk

A clear plastic disk, whose internal dimensions are 3.5 mm in thickness and 81 mm in diameter, is half-filled, and rotates around its axis, placed horizontally. Rotation is obtained with a motor at a speed where the flow on the free surface is continuous $\left(0.04 \mathrm{~s}^{-1}\right)$, with no clamping or jam-


FIG. 20. Segregation in a two-dimensional drum (the drum is 8.1 cm in diameter). The beads are (a) $90 \mu \mathrm{~m}$ with $1 \%$ of 1.5 mm and (b) $150 \mu \mathrm{~m}$ with $0.28 \%$ of 1.5 mm . In both cases, the diameter ratio corresponds to a segregation at an intermediate distance $r$ from the center of the drum.
ming when only filled with small beads. Large beads are 710 $\mu \mathrm{m}, 1.5 \mathrm{~mm}$, or 3 mm in diameter. This is not a true twodimensional (2D) experiment, because the small beads are a lot smaller than the thickness of the disk. The segregation patterns are qualitatively the same as in the 3D drum, and happen for about the same diameter ratios: for a large fraction $(20 \%)$, the large beads are dispatched everywhere inside for high size ratios (above about 7), and stacked at the periphery for small ratios (under about 4). A smaller fraction $(5 \%, 1 \%$, or $0.28 \%)$ leads to intermediate circles of segregation [Figs. 20(a) and 20(b)] for experiments made with $1.5-\mathrm{mm}$ and $710-\mu \mathrm{m}$ large beads. Once again, we observe that while a large bead is moving within the flowing layer, its vertical location compared to the free surface seems constant: a constant fraction of the large bead is visible above the upper surface of the flow. Moreover, its vertical relative position depends on the size ratio: the large beads are 'on the surface," then 'a bit inside the layer," then "half embedded," and eventually 'totally embedded" when increasing the diameter ratio. These observations indicate that the large beads rapidly reach a vertical level and move with the flow at this level, as long as the flow conditions are identical. Quantitatively, the evolution of $r / R$ versus the diameter ratio for a few $1.5-\mathrm{mm}$ large beads in small ones (Fig. 21) is the same as those obtained in the 3D drum (except for the maxi-


FIG. 21. Location of the segregated large beads in the disk $(\boldsymbol{\bullet})$ : $1.5-\mathrm{mm}$ beads represent $0.28 \%$ of the mass, and the small beads are well-sorted beads. We observe the same quantitative evolution as for the 3D drum (see Fig. 14). Due to the geometry, the $r / R$ maximum value is 0.981 .
mum value of $r / R)$. This is compatible with the fact that the structure of the flow is proportional to the size of the drum, and seems to indicate that the stopping distance $r$ is related to this structure. We also noticed that $r / R$ varies slightly with the speed of rotation if a large range of speed is used (from 0.002 to $0.4 \mathrm{~s}^{-1}$ ), but this effect will be dealt with in a further investigation.

Nevertheless, only the experiments with 3-mm large beads could be considered 2D experiments, and we observed that $3-\mathrm{mm}$ beads can act as 'plugs'" and strongly disturb the flow. Indeed, due to the thickness of the disk, small beads cannot easily pass along the sides of a large bead. This phenomenon does not have any effect when the large beads segregate to the periphery, as the large beads roll above the small ones. However, when a 3-mm large bead sinks into the bed and stops on the slope at a distance $r$, partially embedded, it prevents the following small beads from flowing on the free surface. Small beads stack behind this large one, push it, or flow above it, after a period of accumulation. This process greatly modifies the coordinates of the large beads. For high speeds of rotation (about $0.2 \mathrm{~s}^{-1}$ ), the plugs are rapidly buried by the rotation, and the large beads are uniformly distributed in the drum. At lower speed (limit of avalanching, about $0.006 \mathrm{~s}^{-1}$ ), the plugs are sometimes pushed by the main flow, sometimes not. Moreover, the interaction between the large beads is enhanced in the disk: large beads form radial rows (Fig. 22) because they stack behind each other.

Previous studies in a 2D rotating drum [2,3] showed that the locations of one small particle accumulated during several revolutions are the same as the locations of a group of small particles at a given time. Unlike these studies, it does not seem possible to investigate the segregation of a group of large particles by studying the successive trajectories of one large particle in a 2D drum. Moreover, because of the formation of plugs, eventually pushed, the pattern of segregation may not be the same in a 2D system as in a 3D system, even for a single large bead. This observation would need to be checked and studied in a true 2D device, with small and


FIG. 22. Effect of plugs in the segregation in a two-dimensional drum. The large beads are 3 mm , and the small beads are $180-212$ $\mu \mathrm{m}$. At a low rotation speed (about $0.006 \mathrm{~s}^{-1}$ ), there is formation of radial rows of large beads resulting from the accumulation of large beads behind a large bead which stopped on the slope, partially embedded.
large disks of the same thickness as the drum. Because the mass ratio will be related to the size ratio to the power of 2 , and because the packing density and the mechanical strength are modified in a real 2D system, we expect that the evolution of the location of the segregated large disks toward deep levels may occur for larger values of the diameter ratio.

## III. SINKING OF THE LARGE BEADS

The three types of experiments presented in this paper show that large beads can sink inside the deposit instead of segregating at the surface. Comparing the three types of experiments, a visual 'transition' between usual segregation and reverse segregation appears for a size ratio of around 5. However, precise measurements show that this is not a true abrupt transition, and that the location of the segregated large beads evolves continuously from the top toward the bottom of the flow.

In the present study, two effects are involved in an explanation of the segregation. First, a large bead has a large size, and geometrical effects push it to the free surface [1]. Second, a large bead is heavier than a small one, and its mass drives it down to the bottom [8]. Usually the free surface is not located at the bottom of the bed, and these two effects act in opposition. For small size ratios (slightly larger than 1) the geometrical effect is dominant, and the large beads segregate at the surface of the bed. For large size ratios, the mass effect is dominant, and the large beads sink toward the bottom, leading to reverse segregation. Further experiments involving beads of different densities and sizes fully confirm this hypothesis, and will soon be published [28]. It is important to note that the effect of the mass acting against the geometrical effect does not require very large size ratios and is already visible for size ratios as small as 2 (Fig. 14). Actually, these two effects can exactly counterbalance each other in the case of particles with different densities and sizes [4], ending in a homogeneous mixture of the two components. In our experiments, the density is the same for all beads, but their mass is proportional to their size to the power 3 (and


FIG. 23. Because of the value of the packing density in the flow (spacing enhanced on the drawing), or because of the weight of beads located at upper levels, the mechanical strength of the bed (acting against the push-away effect) is strong at the bottom of the flow, and weak at the top. The geometrical effect (dynamical sieving) is efficient when beads are close to each other, i.e., more efficient at the bottom than at the top. Thus the geometrical and pushaway effects vary in opposite ways with the vertical location within the flow, and a large bead moves to the level where the two effects counterbalance.
the pressure due to the mass of a large bead on the bed is proportional to its size). A high mass allows a large bead to push away the other beads, so as to create enough space to go down into the bed. This suggests that the "squeeze expulsion'" term introduced by Savage and Lun [1] could be at least mass dependent, which implies that it is size dependent in the case of particles of the same composition and, perhaps also, friction coefficient dependent [8]. The tendency to sink will increase progressively when increasing the size ratio, as mass and size both vary at the same time; this can explain the continuity of the evolution of $r / R$.

If the two effects of segregation do not counterbalance for a pair of beads, a large bead will go upward or downward and may end up either at the top or bottom of the flow. The packing density of the bed varies from $0 \%$ to about $60 \%$ from the top to the bottom of the flowing phase (Fig. 23). When beads are far from each other, no dynamical sieving occurs; when they are very close no large bead can pass through the network of small beads, and segregation due to the geometrical effect is very efficient. The mechanical strength of the bed (acting against the push-away effect) will also vary with the packing density and with the stress in the flow. If the small beads are far from each other (top of the flow, it is easy to push away some small beads to create some space. If the small beads are close to each other and, furthermore, some pressure is applied onto them due to the weight of the upper layers of beads, a large bead cannot move them away easily. Qualitatively, the geometrical effect is weak at the top of the flow, and strong at its bottom. On the contrary, the push-away effect is efficient at the top and weak at the bottom. This double evolution with the vertical coordinate suggests that for each type of large bead there is an intermediate level where these two effects balance (Fig. 23). The segregation could be a means to study the internal structure of the flowing layer, because a large bead will locate at a level corresponding to the characteristics of the beads and to particular conditions of stress and packing density within the flow.

## A. Influence of the fraction of large beads

The fraction of beads has a strong influence on the segregation patterns. One of the most convincing facts is that the reverse segregation of large beads in a heap is totally masked for every diameter ratio if fractions of large beads as high as $50 \%$ are used. In both heap and drum experiments, segregation switches from reverse to usual when increasing the fraction of large beads. To move downward, a large bead has to push away the other beads. This is easy when the beads of the medium are light (i.e., small), but if the medium contains many large beads, the considered large bead has to push away these heavy beads too. Consequently, large fractions of large beads do not favor the reverse segregation of the large beads. The strength of the medium is controlled by the formation of arches. Let us consider an arch of small beads built onto large beads. Qualitatively, this arch will be less fragile when constituted by only few beads, which is the case for "large" small beads, or "close" large beads. Hence, small diameter ratios, or high fractions of large beads, prevent the large beads from sinking into the bed. We assume that if the large beads constitute a continuous network for the propagation of the stress, it will not be possible for another large bead to penetrate the medium. The sinking or floating of the large particles in or on a bed involves the mechanical strength of the bed and the mass of the large particles.

## B. How to escape segregation?

One of the problems in industry is to mix various components, or to prevent their segregation during their transport as a mixture. A few possibilities come from this study. In a case of a drum mixer, a large size ratio (above 7) and a high enough fraction of large beads (typically 16\%) leads to a relatively uniform distribution of the beads (Figs. 11, 12, and 16). This comes from a combination of the average location of the large beads around about $r / R=0.5$, and the interaction between large beads which spread the distribution throughout the bed. In a case of granular material traveling along a slope, a few large beads, with a size ratio above 5, are distributed inside the bed [Figs. 4(d) and 4(e)], and a mixture with a size ratio 4.3 looks truly homogeneous [Fig. 4(c)]. Counterbalancing equivalent densities and sizes is one possibility to obtain a "homogeneous" mixture after shearing [4]; however, here we propose some conditions which can also be efficient for materials of the same density.

## C. ... and the segregation in a polydisperse medium?

In natural systems, very large blocks are often found at the surface of pyroclastic flows. This observation could appear to be in contradiction with our experimental results because the size ratio between the smallest and the largest particles is much larger than 10 . But a natural medium is made of particles with a wide range of sizes, and not only with two types of beads. Because of the strong dependence of the final location of the beads on the size ratio (from up to down), and without being able to define this main parameter in the case of a continuous distribution of sizes, it is not obvious how to predict where the segregation will drive one particular type of particle.

Some heaps have been made with three types of particles, revealing the importance of the shape of the sizes distribution on the final segregation pattern. For example, (i) a mixture composed of $5 \%$ of $3-\mathrm{mm}$ beads, $5 \%$ of $710-\mu \mathrm{m}$ beads, and $90 \%$ of $300-400-\mu \mathrm{m}$ beads leads to a heap with the $3-\mathrm{mm}$ beads located inside, and $710-\mu \mathrm{m}$ beads forming a ring; and (ii) a mixture composed of $5 \%$ of $3-\mathrm{mm}$ beads, $90 \%$ of $600-800-\mu \mathrm{m}$ beads, and $5 \%$ of $425-\mu \mathrm{m}$ beads gives a heap where $3-\mathrm{mm}$ beads form a ring, and $425-\mu \mathrm{m}$ beads gather in a central core inside the pile. In these two experiments, $3-\mathrm{mm}$ beads can be either inside the pile or in the ring, corresponding to reverse or usual segregation. In these simple cases, a large majority of one of the components allows us to predict the behavior of the beads, by neglecting the third component. However considering a continuous distributions of sizes, it is not possible to reduce the segregation process to a two-component interaction. The segregation in a polydisperse medium will probably depend strongly on the shape of the particle size distribution, and thus this subject needs further investigation.

## IV. CONCLUSION

In three types of experiments, we show that segregation by sizes in a granular medium sheared in a frictional regime does not always drive large beads to the surface of the bed, as previously described. We found that for a mixture of beads of the same density, there is an obvious reverse segregation of the large beads for size ratios above about 5: not even one large bead is on the surface. The previous model of the process of segregation based on the diameters of each bead, and their probabilities of finding a hole of their size, fails to explain these new features of segregation. This implies that some other characteristics (mass, friction coefficient, etc.) are also involved in the segregation process. We show that the effect of the high mass of large beads pushes them down into the bed. For a small fraction of large beads, the segregated beads locate at a particular intermediate level inside the flow. In the three types of experimental device (formation of a heap, flow in a channel, and flow in a halffilled rotating drum), the location of the segregated large beads evolves continuously from the surface to deeper levels when increasing the size ratio. Because of this particular dependence of the location of the segregated beads on the diameter ratio, it will not be possible to easily extrapolate from our study, involving only two types of particles, to the case of segregation in a continuous polydisperse size medium, such as natural avalanche materials.

In our experiments, carried out with beads of the same density and of two different sizes, the intermediate levels of segregated beads are interpreted by the existence of a location within the flow, where the geometrical effect pushing a large bead up to the free surface and the mass effect pushing it down to the bottom exactly counterbalance. The segregation is the result of a balance between various effects, and not as a squeezing out of one type of particle out of the main bed. Consequently, for beads of different densities, the segregation could place the large beads at any intermediate level depending on their mass and size. We also show that working with large fractions of large beads can lead to 'homogeneous'" mixtures depending on the vertical coordinate of the
mean level where the segregated beads locate. Adjusting the size and fraction of the components could be a means to prevent the segregation or to mix the components. This method could be extended to the case of particles of different densities, leading to a whole range of possibilities to prevent segregation in granular media. Moreover, as the local strength of the bed and the geometrical effect depend on the structure of the flow, these segregated bead levels will possibly appear at different heights during some other regimes
of flow, and the segregation could be a means to study the stress field and the internal structure of the granular flows.

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